

Small Scale Density Inhomogeneities In The Equatorial Spread-F Ionosphere

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Abstract: High frequency radar backscatter experiments have revealed a spectrum of short wavelength irregularities (i.e., 3m, 1m, 36cm and 11cm) in the Equatorial Spread-F [ESF] Ionosphere. Vertical drift of metallic ion species from meteoric ablation regions (90-110 km) further make ESF plasma dynamics complex. In context with the smallest of all the observed irregularities i.e. 11 cmin the ESF region, kinetic lower hybrid drift instability has been invoked to account for observations even well below ion gyroradius. Ion dynamics has been considered separately for both O⁺ and Fe⁺ ionic species whereas Bhatnagar – Gross - Krook collisional effects have been incorporated in the electron dynamics. Further, frequency and growth rate estimates have been discussed in light of kinetic hybrid drift instability adopted earlier to account the unified spectrum of small scale irregularities.

Keywords: plasma waves and instabilities, nonlinear wave-particle interaction, plasma turbulence

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I. Introduction:

High frequency radar backscatter experiments have revealed a spectrum of short wavelength irregularities (i.e., 3m, 1m, 36cm and 11cm) in the Equatorial Spread-F [ESF] Ionosphere[1-2]. EXB vertical drift of metallic ion species from meteoric ablation regions (90-110 km) further make ESF plasma dynamics more complicated with additional ionospheric concentration in the ionosphere[3]. In context with the smallest of all the observed irregularities i.e. 11 cmin the ESF region, kinetic lower hybrid drift instability has been invoked to account for observations even well below ion gyro-radius which takes into account two different species ionic composition. Ion dynamics has been considered separately for both O⁺ and Fe⁺ ionic species whereas Bhatnagar – Gross - Krook collisional effects have been incorporated in the electron dynamics to account the additional features in the analytical model[4-5].

II. On 11cm Equatorial Spread-F Irregularities using Two ion species model:

In this paper, we will invoke a two ion species explanation of the 11 cm irregularities during Equatorial Spread F[6]. The assumptions are as follows:

[1]. Let us consider a plasma immersed in a homogeneous, unidirectional field $\mathbf{B} = B_0 \hat{\mathbf{e}}_z$ with an inhomogeneous density profile $n = n(x)$ and has a constant temperature $T_e = T_i = T$.

[2]. Each species α [electrons and O⁺ ions] has a diamagnetic drift velocity $\mathbf{V}_{d\alpha} = \left(\frac{V_\alpha^2}{2\Omega_\alpha} \right) \frac{d \ln n}{d\mathbf{x}}$.

Here, $V_\alpha = \left(\frac{2T_\alpha}{m_\alpha} \right)^{1/2}$ is the thermal velocity, $\Omega_\alpha = \left(\frac{e_\alpha B_0}{m_\alpha c} \right)$ is the cyclotron frequency and

$\mathbf{n} = \mathbf{n}_e \approx \mathbf{n}_i$ is the condition of quasineutrality.

[3]. A net current exists in the plasma given as $\mathbf{J}_0 = \mathbf{en}(\mathbf{V}_{di} - \mathbf{V}_{de}) \hat{\mathbf{e}}_y \approx 2en\mathbf{V}_{di} \hat{\mathbf{e}}_y$ and it provides the free energy to drive the instability.

[4]. Perturbed quantities are assumed to vary as $\exp[\mathbf{i}k\mathbf{y} - \mathbf{i}\omega t]$.

[5]. Only electrostatic oscillations are considered since $\beta \ll 1$.

[6]. The ions behave as unmagnetised particles for the wavelengths under consideration

[$k r_{Li} \sim 10^2$] because of ion-ion collisions.

[7]. Electron-neutral, electron-electron and electron-ion collisions are included in the analysis via an electron collision frequency $\nu_e = \nu_{en} + \nu_{ei} (1 + 0.3\mathbf{b}_e)$, where $\mathbf{r}_{Le} = \frac{\mathbf{v}_e}{|\Omega_e|}$ is the mean

electron Larmor radius and $\mathbf{b}_e = \frac{\mathbf{k}^2 \mathbf{r}_{Le}^2}{2}$.

[8]. Local approximation has been used which requires $\mathbf{kL}_n \gg 1$, where $\mathbf{L}_n = \left(\frac{d \ln n}{d\mathbf{x}}\right)^{-1}$ is the density gradient scale length.

Following these assumptions and the equilibrium

$$\mathbf{F}_{i0} = \mathbf{n}_0 \left(\frac{1}{\pi \mathbf{V}_i^2}\right)^{1/2} \exp \left[\frac{-\left\{ \mathbf{v}_x^2 + (\mathbf{v}_y - \mathbf{V}_{di})^2 + \mathbf{v}_z^2 \right\}}{\mathbf{V}_i^2} \right]$$

the equation for the ion susceptibility [Huba et. al. 1978][Huba and Ossakow, 1981] as

$$\chi_i = \frac{2\omega_{pi}^2}{\mathbf{k}^2 \mathbf{V}_i^2} \left[1 + \frac{\omega - \mathbf{kV}_{di}}{\mathbf{kV}_i} z \left(\frac{\omega - \mathbf{kV}_{di}}{\mathbf{kV}_i} \right) \right] \quad [1]$$

Expanding the plasma dispersion z in a small argument limit $z(\psi) \approx i(\pi)^{1/2}$, the ion susceptibility for the single species comes out to be

$$\chi_i = \left(\frac{1}{\mathbf{k}^2 \lambda_{di}^2}\right) \left[\frac{\mathbf{kV}_i + i\pi^{1/2}(\omega - \mathbf{kV}_{di})}{\mathbf{kV}_i} \right] \quad [2]$$

If we consider the ion susceptibilities for the two ions separately for O^+ and Fe^+ ionic species, then we have

$$\chi_{i1} = \left(\frac{1}{\mathbf{k}^2 \lambda_{di1}^2}\right) \left[\frac{\mathbf{kV}_{i1} + i\pi^{1/2}(\omega - \mathbf{kV}_{di1})}{\mathbf{kV}_{i1}} \right] \quad [3]$$

$$\chi_{i2} = \left(\frac{1}{\mathbf{k}^2 \lambda_{di2}^2}\right) \left[\frac{\mathbf{kV}_{i2} + i\pi^{1/2}(\omega - \mathbf{kV}_{di2})}{\mathbf{kV}_{i2}} \right] \quad [4]$$

where χ_{i1}, χ_{i2} are the ion susceptibilities of the two ions taken into consideration.

and $\mathbf{V}_{i1}, \mathbf{V}_{i2}$ are their respective thermal velocities whereas $\lambda_{di1}, \lambda_{di2}$ are their individual debye lengths. Similarly, for the electron susceptibility [6], we rewrite

$$\chi_e = \frac{2\omega_{pe}^2}{\mathbf{k}^2 \mathbf{V}_e^2} \left[1 - \left(\frac{\omega - \mathbf{kV}_{de} + i\nu_e}{\omega + i\nu_e} \right) \Gamma_0 \right] \left[1 - \left(\frac{i\nu_e}{\omega + i\nu_e} \right) \Gamma_0 \right]^{-1} \quad [5]$$

Now writing electron susceptibility including substitution terms of two ions, then the electron susceptibility will come out to be

$$\chi_e = \left(\frac{1}{\mathbf{k}^2}\right) \left\{ \frac{1}{\lambda_{di1}^2} + \frac{1}{\lambda_{di2}^2} \right\} \frac{\left(\frac{\mathbf{T}_i}{\mathbf{T}_e}\right) [(\omega + i\nu_e)(1 - \Gamma_0) + \mathbf{kV}_{de}\Gamma_0]}{\omega + i\nu_e(1 - \Gamma_0)} \quad [6]$$

Here $\Gamma_0 = \mathbf{I}_0(\mathbf{x})\mathbf{e}^{-\mathbf{x}}$ and \mathbf{I}_n is the modified Bessel function of order n Equation [6] is based upon the Bhatnagar-Gross-Krook collision model but does not correctly treat electron-electron collisions [Rukhadze and Silin, 1968]. However, electron viscosity is approximately modeled via the term proportional to $\mathbf{k}^2 \mathbf{r}_{Le}^2$ [Mikhailovskii and Pogutse, 1966],[Huba and Ossakow, 1981].

Since the 11 cm irregularities correspond to $\mathbf{k}\mathbf{r}_{Le} \approx 2$, it is clear that electron viscous effects are only moderately important. Moreover, the Bhatnagar-Gross-Krook model is adequate in the absence of temperature gradients [Rukhadze and Silin, 1968] which is the situation prevalent in the F region. Thus, equation [6] can be used properly to describe the electron response qualitatively. However, the quantitative results based on equation [6] are approximately correct, since a model Fokker Planck equation is used to describe the collisionality. As $\mathbf{V}_{di} \ll \mathbf{v}_i$ [which corresponds to $\mathbf{r}_{Li} \ll \mathbf{L}_n$], the plasma dispersion function can be expanded in the small argument limit $\mathbf{z}(\psi) \approx \mathbf{i}(\pi)^{1/2}$. The dispersion relation for the lower hybrid drift instability for the case of double ions in the same analogy as single ion [6] can be written as,

$$\mathbf{D}(\omega, \mathbf{k}) = 1 + (\chi_{i1} + \chi_{i2}) + \chi_e = 0 \tag{7}$$

$$1 + \left(\frac{1}{\mathbf{k}^2 \lambda_{di1}^2} \right) \frac{[\mathbf{k}\mathbf{V}_{i1} + \mathbf{i}\pi^{1/2}(\omega - \mathbf{k}\mathbf{V}_{di1})]}{\mathbf{k}\mathbf{V}_{i1}} + \left(\frac{1}{\mathbf{k}^2 \lambda_{di2}^2} \right) \frac{[\mathbf{k}\mathbf{V}_{i2} + \mathbf{i}\pi^{1/2}(\omega - \mathbf{k}\mathbf{V}_{di2})]}{\mathbf{k}\mathbf{V}_{i2}} \tag{8}$$

$$+ \left(\frac{1}{\mathbf{k}^2} \right) \left\{ \frac{1}{\lambda_{di1}^2} + \frac{1}{\lambda_{di2}^2} \right\} \frac{\left(\frac{\mathbf{T}_i}{\mathbf{T}_e} \right) [(\omega + \mathbf{i}\mathbf{v}_e)(1 - \Gamma_0) + \mathbf{k}\mathbf{V}_{de}\Gamma_0]}{\omega + \mathbf{i}\mathbf{v}_e(1 - \Gamma_0)} = 0$$

Here, the value of both O^+ and Fe^+ ionic species susceptibilities and electron susceptibilities have been kept from equations [3] [4] and [6] in the above equation. Now setting $\omega = (\omega_r + \mathbf{i}\gamma)$ in the above equation and applying the limit $\gamma \sim \mathbf{v}_e \ll \omega_r$, we obtain the oscillation frequency as

$$\omega_r = \Gamma_0 \mathbf{k} \left[\mathbf{V}_{d1} \left(\frac{\mathbf{n}_1}{\mathbf{n}_1 + \mathbf{n}_2} \right) + \mathbf{V}_{d2} \left(\frac{\mathbf{n}_2}{\mathbf{n}_1 + \mathbf{n}_2} \right) \right] \left[1 + \frac{\mathbf{k}^2}{\left(\frac{1}{\lambda_{di1}^2} + \frac{1}{\lambda_{di2}^2} \right)} + \frac{\mathbf{T}_i}{\mathbf{T}_e} (1 - \Gamma_0) \right]^{-1} \tag{9}$$

When the contribution due to second ion vanishes, we get the corresponding oscillation frequency for the single species case as,

$$\omega_r = \Gamma_0 \mathbf{k}\mathbf{V}_{di} \left[1 + \mathbf{k}^2 \lambda_{di}^2 + \left(\frac{\mathbf{T}_i}{\mathbf{T}_e} \right) (1 - \Gamma_0) \right]^{-1} \tag{10}$$

which is exactly the same as that of oscillation frequency obtained by Huba et al 1981[6-8]. Lastly, we obtain the growth rate in term of two ion species as

$$\gamma = -\omega_r \left[\frac{\omega_r}{\mathbf{k} \left[\mathbf{V}_{d1} \left(\frac{\mathbf{n}_1}{\mathbf{n}_1 + \mathbf{n}_2} \right) + \mathbf{V}_{d2} \left(\frac{\mathbf{n}_2}{\mathbf{n}_1 + \mathbf{n}_2} \right) \right]} \right] \tag{11}$$

$$\left[\frac{\pi^{1/2}}{\Gamma_0} \left[\frac{(\omega_r - \mathbf{k}\mathbf{V}_{di1})}{\mathbf{k}\mathbf{V}_{i1} \left(1 + \frac{\lambda_{di1}^2}{\lambda_{di2}^2} \right)} + \frac{(\omega_r - \mathbf{k}\mathbf{V}_{di2})}{\mathbf{k}\mathbf{V}_{i2} \left(1 + \frac{\lambda_{di2}^2}{\lambda_{di1}^2} \right)} \right] + \frac{\mathbf{T}_i}{\mathbf{T}_e} \frac{\mathbf{v}_e}{\omega_r} (1 - \Gamma_0) \right] \left\{ 1 + \frac{\mathbf{k} \left[\mathbf{V}_{d1} \left(\frac{\mathbf{n}_1}{\mathbf{n}_1 + \mathbf{n}_2} \right) + \mathbf{V}_{d2} \left(\frac{\mathbf{n}_2}{\mathbf{n}_1 + \mathbf{n}_2} \right) \right]}{\omega_r} \left(\frac{\mathbf{T}_e}{\mathbf{T}_i} \right) \right\}$$

When the contribution due to second ion vanishes, we get the corresponding growth rate for the single ion species case as,

$$\gamma = -\omega_r \left(\frac{\omega_r}{\mathbf{kV}_{di}} \right) \left[\frac{\pi^{1/2}}{\Gamma_0} \frac{(\omega_r - \mathbf{kV}_{di})}{\mathbf{kV}_i} + \frac{\mathbf{T}_i \mathbf{v}_e}{\mathbf{T}_e \omega_r} (1 - \Gamma_0) \left\{ 1 + \frac{\mathbf{kV}_{di} \mathbf{T}_e}{\omega_r \mathbf{T}_i} \right\} \right] \quad [12]$$

where, $\lambda_{di}^2 = \frac{\mathbf{v}_i^2}{2\omega_{pi}^2}$ and the argument of Γ_0 has been suppressed.

In the absence of collisions ($\mathbf{v}_e = 0$), instability occurs for

$$\omega_r < \mathbf{k} \left[\mathbf{V}_{d1} \left(\frac{\mathbf{n}_1}{\mathbf{n}_1 + \mathbf{n}_2} \right) + \mathbf{V}_{d2} \left(\frac{\mathbf{n}_2}{\mathbf{n}_1 + \mathbf{n}_2} \right) \right]$$

and there is no threshold requirement.

However, electron collisions are stabilizing and place a threshold condition on the drift velocity to excite the mode [6]. From substitution, the above equations yield the critical drift velocity [i.e. such that $\gamma > 0$] given by

$$\left[\frac{\mathbf{V}_{d1} \left(\frac{\mathbf{n}_1}{\mathbf{n}_1 + \mathbf{n}_2} \right) + \mathbf{V}_{d2} \left(\frac{\mathbf{n}_2}{\mathbf{n}_1 + \mathbf{n}_2} \right)}{\mathbf{v}_i} \right]_{cr} > \left[\frac{\frac{\mathbf{v}_e}{\Omega_i} \left(\frac{\mathbf{m}_e}{\mathbf{M}_1 + \mathbf{M}_2} \right)^{1/2} \left\{ 1 + \frac{\mathbf{n}_2}{\mathbf{n}_1} \left(\frac{\mathbf{M}_2}{\mathbf{M}_1} \right)^{1/2} \right\}}{\frac{1}{\pi^{1/2}} \mathbf{kr}_{Le} \Gamma_0} \cdot \frac{\left\{ 2 + \frac{\mathbf{k}^2}{\left(\frac{1}{\lambda_{d1}^2} + \frac{1}{\lambda_{d2}^2} \right)} - \Gamma_0 \right\}^2 \left(2 + \frac{\mathbf{k}^2}{\left(\frac{1}{\lambda_{d1}^2} + \frac{1}{\lambda_{d2}^2} \right)} \right)}{2(1 - \Gamma_0) + \left(2 + \frac{\mathbf{k}^2}{\left(\frac{1}{\lambda_{d1}^2} + \frac{1}{\lambda_{d2}^2} \right)} \right)} \right]^{1/2}$$

Again, vanishing the contribution due to second ion, we get the corresponding growth rate for the single ion species case as,

$$\left(\frac{\mathbf{V}_{di}}{\mathbf{v}_i} \right)_{cr} > \left[\frac{\mathbf{v}_e}{\Omega_i} \left(\frac{\mathbf{m}_e}{\mathbf{m}_i} \right)^{1/2} \frac{1}{\pi^{1/2}} \frac{(1 - \Gamma_0) (2 + \mathbf{k}^2 \lambda_{di}^2 - \Gamma_0)^2 (2 + \mathbf{k}^2 \lambda_{di}^2)}{\mathbf{kr}_{Le} \Gamma_0 \cdot 2(1 - \Gamma_0) + \mathbf{k}^2 \lambda_{di}^2} \right]^{1/2}$$

which is same as obtained by Huba et al, [6] for the case of single ion.

Above Equation corresponds to a critical density gradient scale length via

$$\mathbf{L}_n^{cr} < \left[\frac{\mathbf{v}_1 \left(1 + \frac{\lambda_1^2}{\lambda_2^2} \right) + \mathbf{v}_2 \left(1 + \frac{\lambda_2^2}{\lambda_1^2} \right)}{\mathbf{V}_{d1} \left(\frac{\mathbf{n}_1}{\mathbf{n}_1 + \mathbf{n}_2} \right) + \mathbf{V}_{d2} \left(\frac{\mathbf{n}_2}{\mathbf{n}_1 + \mathbf{n}_2} \right)} \right]_{cr} \left(\frac{\mathbf{r}_{Li}}{2} \right)$$

The above expression reduces to the single ion case of Huba et al, [6] as $\mathbf{L}_n^{cr} < \left(\frac{\mathbf{v}_i}{\mathbf{V}_{di}} \right)_{cr} \left(\frac{\mathbf{r}_{Li}}{2} \right)$

The expressions obtained here using the technique of Two ion species into consideration are in good agreement to the single ion species treatment of Huba and Ossakow, [1981]. At this point, we discuss the physical insight obtained in the light of single ion treatment for the observation of 11 cm irregularities during Equatorial Spread F.

We consider the parameters typical to the Equatorial Spread F [Huba et al 1981] as listed under:-

$$\mathbf{B} = 0.3\mathbf{G}, \quad \mathbf{T}_e = \mathbf{T}_i = 0.1\mathbf{eV} \quad \text{and} \quad \mathbf{m}_i = 16\mathbf{m}_p,$$

The collision frequency being given by

$$\mathbf{v}_e = \mathbf{v}_{en} + \mathbf{v}_{ei} \left(1 + 0.15 \mathbf{k}^2 \mathbf{r}_{Le}^2\right)$$

where

$$\mathbf{v}_{en} = 5.0 \times 10^{-8} \mathbf{n}_n \mathbf{T}_e^{1/2} \mathbf{s}^{-1}$$

$$\mathbf{v}_{ei} = \left(\frac{\lambda}{3.5} \times 10^5\right) \left(\frac{\mathbf{n}_e}{\mathbf{T}_e^{3/2}}\right) \mathbf{s}^{-1}$$

and $\lambda = 23.4 - 1.15 \log \mathbf{n}_e + 3.45 \log \mathbf{T}_e$

Here, \mathbf{n}_n is the neutral density, \mathbf{n}_e is the electron density, and \mathbf{T}_e is given in eV.

Using the parameters listed above, we apply it to the above expressions for Two ion species technique. Further, the results turn out to be in good agreement with the single ion treatment of Huba et al 1981, which supports our intention that two ion species technique can be equally applied to the 11 cm irregularities of the Equatorial Spread F in a similar fashion as the single ion technique of Huba et al 1981 valid for altitudes 500 and 250 km observed for radar backscatter measurements.

III. Conclusion:

As Equatorial Spread F develops, density depletions rise to the topside of the F region where the neutral density is low. Within these plasma bubbles [density depletion regions, electron collisional effects are minimum], sharp density gradients exist which can excite the lower hybrid drift instability using two ion species method. The density fluctuations associated with the instability give rise to the radar backscatter measurements at 1m, 36 cm and 11 cm.

Thus, based upon the linear theory of the lower hybrid drift instability, it is found that this mode is the most probable cause of the small scale irregularities [$\leq 1\text{m}$] observed during Equatorial Spread F. It turns out that several interesting features of the two ion species model arises out of this analytical treatment. This indicates that for a given density and density gradient scale length, only a certain range of irregularities can be excited linearly. Further, larger values of the neutral density require larger drift velocities [or shorter density gradient scale lengths] as expected. The role of this instability, frequency and growth rate estimates have been discussed in light of kinetic hybrid drift instability [4-5] adopted earlier to account the unified spectrum of small scale irregularities.

References:

- [1]. Kelley, Michael C., Jonathan J. Makela, Odile de La Beaujardière, and John Retterer, *Reviews of Geophysics* 49, no. 2 (2011).
- [2]. Kelley, Michael C. *The Earth's Ionosphere: Plasma Physics & Electrodynamics*. Vol. 96. Academic press, (2009).
- [3]. Huba, J. D., J. Krall, and G. Joyce, *Geophysical Research Letters* 36.10 (2009).
- [4]. Bose, M. "Ionospheric F-layer small-scale irregularities: a possible explanation." *Plasma Science, 2004. ICOPS 2004. IEEE Conference Record-Abstracts. The 31st IEEE International Conference on. IEEE, 2004.*
- [5]. BOSE, M., and YS SATYA. "Explanation of small scale irregularities of F region by ion-ion hybrid drift wave." *Indian Inst. of Tech, The 1994 International Conference on Plasma Physics, combined with the 6th Latin American Workshop on Plasma Physics* p 210-213 (SEE N 96-22762 07-75). 1994.
- [6]. Huba, J. D., and S. L. Ossakow, *Journal of Geophysical Research: Space Physics*, 86.A2: 829-832 (1981).
- [7]. Y.S. Satya, Personal communication, (1990).
- [8]. J.K. Atul, Ph.D. Thesis, 2013.

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